

# AN ABSENCE AND EMERGENCE OF COLLECTIVITY IN THE MATHEMATICS CLASSROOM<sup>1</sup>

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## **Abstract**

*In a general sense, collectives as learning systems have been conceived as arising through complex, creative, and emergent processes. Such claims necessitate research that observes in detail, how these processes are evidenced in the moment-to-moment interactions of students in the classroom. This paper responds to this call, by examining both the absence and emergence of collectivity that results as three learners work on a geometric task.*

## **Introduction**

Well established in mathematics education is research that conceptualizes collectives as complex learning systems. Defined as arising out of the mathematical interaction of individuals, collectives are viewed as evolving within social-cultural contexts (Bowers & Nickerson, 2001; Cobb, 1999). Any collective then, possesses qualities that are inherent but not identical to its individual members. Moreover, as a larger cognitive system, a collective is not simply a collection of individuals but a cohesive entity with its own integrity (Bateson, 1972) that too affects the kinds of experiences and mathematical ways of knowing that arise in the classroom. In the same manner that an individual's conceptualizations or "structure" (Maturana and Varela, 1987) defines what that person thinks or does, it is the coordination of the members' activity (that is grounded in their individual and collective understanding) that defines a collective and determines whether and how it functions as a coherent unity. From a complexity science perspective, collectives cannot be achieved by simply grouping individuals together or through the contributions of single agents. Instead, it is through the fluid, spontaneous interactions and coordinated responses of one individual to another that enables coherence amongst individuals and gives rise to a collective.

Davis and Simmt (2003) identify five interdependent conditions that they conceive as necessary for the emergence of collectives and collective activity in the classroom: *internal diversity, redundancy, decentralized control, organized randomness, and neighbor interactions*. Internal diversity refers to the intelligence of a system and its ability to respond appropriately or innovatively in a given situation. Redundancy concerns the similarities such as common experiences, assumptions, and language that

exist amongst the members of a collective. These shared qualities create the coherence and stability necessary for members to interact with one another as well as to compensate for one another's shortcomings. Decentralized control entails that knowledge is distributed across agents and collective activity emerges from the coordinated actions of the individual members. Organized randomness refers to the rich conceptual possibilities that arise when constraints within contexts are neither too redundant nor diverse. And in relation to this, the authors describe neighbor interactions as that which results when there is an abundance of conceptual possibilities and the ideas and understandings that emerge are made to "stumble across one another" (p. 156).

A survey of recently published books, journal articles, and conference proceedings in education reveals an ever increasing body of research that documents the presence of these theorized conditions as well as ways in which to foster these in order to expand the possibilities for collectives to occur within different learning environments. While the theoretical discourse of complexity science together with the empirical studies that are framed by it characterize groups of learners as dynamic and self-organizing entities, there is a need for in-depth studies that examine the very emergence of collectives; that is, what are the dynamic and self-organizing moment-to-moment manners in which collective unities arise? This paper responds to this call by examining three learners as they worked on a geometric task.

### **Study Context**

The school in which this study takes place is located near the university. Mrs. Wiley, the regular teacher is also the school's vice principal. She collaborated with the research team in designing the curriculum. It is important to note that while the five conditions outlined by Davis and Simmt were inherent in our planning of the lessons, these considerations were in no way used as a formulaic approach to design the learning settings or implemented with the goal of "making" collectives happen in the classroom. Rather, lessons were designed so as to accommodate for a range of conceptual ways in which the students might choose to engage mathematically as individuals or collectives. The teaching of these lessons was a shared effort whereby Mrs. Wiley and members of the research team assumed either role of a lead, team, or supporting teacher.

The episode for this study is from a lesson that occurred during the second week of the class exploring geometric concepts. The class was divided into groups of three and each group was given a box containing a "mystery object." Sonia, Jade, and Maeve-- the students featured in this paper, constitute one of seven working groups. Miss Jang, a research assistant, provided teaching support to the students. The students put their hand, one at a time, into the box as many times as they wished to touch the object that they could not see. This task is similar to the experiment conducted by Piaget and Inhelder

(1956) in that it involves the children working as individuals and tactilely exploring unseen objects. Differently, instead of drawing a picture of the unseen object or matching it with a similar visually accessible object, the children in this study were to create a plasticine model of the object they felt in the box, agree as a group on the identity of the object, and provide reasons for their conclusion. Below is a transcript of the conversation that happened just after they had each taken a turn and put their hand into the box to touch the mystery object.<sup>2</sup>

(2:50)

01 Maeve: Do you know what it is? I do. It's a cube . . . smooth.

02 Jade: [makes a questioning grimace]

03 Sonia: No it's not a cube [shakes her head while touching the object].

(2:51)

04 Jade: I didn't feel it as a cube.

05 Sonia: Me either.

06 Maeve: I did. I checked the sizes, the flat.

(3:02)

07 Jade: [puts her hand back into the box.]

08 Sonia: You should feel around.

09 Jade: [puts her hand in the box] If I feel the top, it seems like it's square but if I feel the side, it seems like rectangle.

10 Maeve: I don't feel any rectangle. I am making mine square.

(3:25)

11 Jade: I think it is oblong.

12 Maeve: I am making mine a cube . . . if it's the same.

13 Sonia: I think it's this [holds up her rectangular prism].

Touch it. See the flat parts. If you touch like this, you will see.

14 Maeve: I think it's this, a cube [puts her hand back into the box].

### **Initial Observations and Focus of Inquiry**

Just over three minutes into the lesson, it is evident from the students' conversation and their plasticine models that each child has established for herself what the mystery object is, and interestingly, there are two different outcomes. Jade and Sonia on their own, have made a rectangular prism with two square and four narrow oblong faces and Maeve, has made a cube. As the conversation takes shape, moving from student to student, the emergence of particular ideas that eventuated these two outcomes may be observed as Sonia, Jade, and Maeve make their personal conceptions available to one another through mathematical argumentation. Also noticeable, is the individual nature of the students' conceptions that are marked by "I" statements which they consistently use as they

articulate their perceptions and related interpretations.

Yet, examining the conversation through inquiring into how the students transact with one another, not only is the emergence of geometric ideas and individual conceptualizations about the mystery object observable, but simultaneously on a social level, the evolution of two different patterns of flow between the students' ideas and conceptions comes into focus—namely, one pattern of flow that eventuates the absence of collectivity and another that eventuates the emergence of it. In the next two sections, we identify and interrogate the ideas, conceptualizations, and the dynamics inherent in these patterns to develop an understanding for the events that they occasioned.

### **An Absence of Collectivity**

It is evident from reading the transcript of the students' conversation that because Maeve conceives the mystery object as something different than what her peers identify it to be, this is what precludes any kind of collectivity from arising between Maeve and her peers. However, not so obvious and requiring further examination is an understanding of the particulars with respect to the students' transactions with each other and how these seemingly subtle events evolved and occasioned the absence of collectivity between Maeve and her classmates.

At the beginning of the lesson, Maeve (turn 6) and Sonia (turn 13) both talk about the object as possessing "flat" parts. Maeve reveals what she means by "flat" in the first minutes of the lesson where through her continual responses to the other students, explains that the sizes of the flat parts of the mystery object are the same and that they are square. However, it is not until after this episode and another equal period of time has passed that Sonia reiterates to Miss Jang, a support teacher, that the object still has "flat faces" and this time, Sonia articulates in a visible way, exactly what she means. She straightens her right thumb and index finger while holding them closely together and parallel to each other. Pointing to one of the oblong faces on her rectangular prism, she runs her right index and second finger along its edges. Sonia then holds one hand flat against each of the two square faces on her model and adds, "and it has squares here." Sonia's verbal, gestural, and physical actions explicate that she is not talking about squares at all when she says, "flat faces" but rather, oblongs.

What appears at first, to be a common or redundant term that both students use to describe the mystery object, upon closer review and much later, turns out to have two diverse meanings. Unlike Maeve's enactions that locate the term as referring to square faces, when Sonia holds up her model (turn 11), she only indicates that it has flat parts. She does not identify where these flat parts actually exist on the model and thus, disables any opportunity to differentiate between the two conceptions. We conceive that as far as Maeve was concerned, it is likely that Sonia's use of the term did not add anything to what Maeve already knew about the mystery object. Further still, even when there is

mention of rectangles (turn 9) or oblongs (turn 11) by Jade, because Maeve previously “checked the sizes” (turn 6) and found them to be “the same” (turn 12), she knows that the object’s flat faces have to be squares and therefore, it is impossible for the object to have any oblongs or be anything other than a cube.

It is these conceptual and temporal contingencies that we see as contributing to the incoherence that arises in the children’s transactions which then creates an absence of collectivity amongst Maeve and her peers. Put another way, it is Maeve, Sonia, and Jade’s personal conceptions or, “structures” (Maturana and Varela, 1987) that allow them to initially bring the mystery object into being as a plasticine cube or a rectangular prism. However, it is when the students transact on a social level with each other that what exists as coherent parts within their individual structures (i.e., Maeve’s perception of squares and the other two student’s perception of square and oblongs) now become structural discrepancies that do not allow for any kind of collective coordination or, social coupling to take place in Maeve’s activity with her peers.

### **An Emergence of Collectivity**

In turning our focus to Sonia and Jade, we see from the transcript that they both conceived the mystery object as having the same identity. Given this and the fact that they created their models on their own, we surmise that Sonia and Jade independently came to the same conclusion that the mystery object was a rectangular prism with two squares and four oblongs.

Curiously though, when we focused on the movement of Sonia and Jade’s ideas and conceptions in the conversation, we noticed a definite coherence that evolved through their ongoing transactions. Below, we describe how this coherence between their individual conceptualizations developed by identifying the geometric ideas that arose (in **bolded** text) and the physical and verbal manners in which Sonia and Jade made their conceptions of these ideas available to each other.

Jade’s grimace (turn 2) that follows Maeve’s claim of the mystery object as a cube provides a nonverbal response that indicates **the object is not a cube**. Both of Sonia’s actions of shaking her head back and forth while touching the object and replying, “No. It’s not a cube” (turn 3) are observed as her recognition and corroboration of Jade’s assertion that **the object is not a cube**. Further still, when Jade states “I didn’t feel it as a cube” (turn 4) and Sonia immediately backs this up with, “me either,” (turn 5) the two students establish for a third time, that **the object is not a cube**. Sonia’s next suggestion that Maeve “should feel around” (turn 8) presents the idea that **the object has features that are different from a cube** while Jane’s response of putting her hand in the box, moving it about, and stating **the top of the object had a different shaped face (i.e., square) than its side (i.e., oblong)** (turn 9) **or, oblong** (turn 11) connect and elaborate on

Sonia's previous suggestion. Towards the end of the conversation when Sonia announces, "I think it's this" and reveals her model of **a rectangular prism with two squares and four oblongs** (turn 13), this physical articulation of the mystery object expresses both Jade's and her conceptions as a unified whole. Specifically, the mystery object is **not** (a) **a cube but** (b) **a rectangular prism** (c) **composed of two square faces and** (d) **four oblong faces that are** (e) **perpendicularly adjacent to them.**

In these recurrent transactions, we see Sonia and Jade coordinating and re-coordinating their personal conceptions of the mystery object. This coordination however, does not arise through a process of negotiation and establishing of a common set of knowledge that exists between them. But rather, through their ongoing transactions, their personal conceptions about particular ideas coalesce as an integrated and coherent system of knowing that is immanent in the two of them as a collective unity. For Jade and Sonia, their personal conceptualizations of the mystery object self-organize by clustering and connecting to common geometric ideas that arise as they make their understandings available to the other students. Sonia and Jade's collectivity is in essence, the coherence that evolves between their personal conceptions of the mystery object and the geometric ideas evoked by them. Even though Sonia and Jade reveal differences or diversity in their personal understandings of the mystery object there is importantly, similarities or redundancies which they recognize and around which their conceptions self-organize. Their collective comprehension is co-evolutionary and this is reflected in how their actions progress recursively as responses made in relation to previous transactions.

We conceive that Sonia and Jade coalesced and became a collective unity from this point of the students' conversation onward. We believe this to be true as we see characteristics that indicate Sonia and Jade's collectivity throughout the rest of the lesson (which runs for another twenty minutes). In our discussion that follows, we occasionally draw on later parts of the lesson; not to examine the collective activity of Sonia and Jade, but to substantiate our claim on the emergence and presence as a collective.

In general, we observed noticeable changes in how Sonia and Jade proceeded to talk and work after Sonia held up her plasticine model. Specifically, we noticed that Sonia and Jade ceased to use "I" statements when verbally articulating their conceptions in the conversation and instead, they either used "we" or did not use any pronoun at all when making statements about the mystery object. This transformation in their speech, suggests that in these instances, Sonia and Jade were working as a collective; that is, by choosing not to use "I" but "we" or no pronoun at all, indicates that the students did not consider the conceptualizations that they presented as belonging to either one of them but rather, existing mutually or distributed across them. This contrasts with what we hear in Maeve's speech as the lesson proceeds. In keeping with the first minutes of the lesson,

Maeve continues to use exclusively “I” statements when she talks. Clearly for Maeve, she identifies the ideas and conceptions that she presents in the lesson as belonging to specifically to her.

Another transformative change that we noticed as the lesson progressed was in *how* Sonia and Jade conducted their work. This shift that we observed concerns the nature of Sonia and Jade’s activity-- more specifically-- the intent of the students’ actions in which the pair’s verbal and physical actions take on a pedagogical function. This shift is made obvious in their verbal statements to Maeve. For example, when Sonia says, “If *you* touch it like this, *you* will see” (turn 13), or in latter parts of the lesson when Jade says, “*let’s* try and find the long face, so you can feel the square face” (approx. 7:11), or when Sonia says, “Look what *we* can do” (approx. 15:35). Instead of responding to Maeve’s expressed conceptualizations of the object as a cube with individual knowings that press against hers, Jade and Sonia proceed to work in tandem and in ways that we as observers, recognize as collective actions directed towards prompting new experiences for Maeve and from which she proceeds to make new sense of the mystery object.

### **Reflections on the Co-existence of the Absence and Emergence of Collectivity**

What appeared to be a straightforward conversation amongst three elementary students turned out to be a complex space in which to explore and interrogate both the absence and the emergence of the students’ collectivity. Prompting our curiosity a little more, we were compelled to ask, what else might we learn about collectives by reflecting on the simultaneous absence and emergence of the students’ collectivity in this lesson.

First, this geometry lesson convincingly exemplifies the point: If we conceive in a theoretical way that collectives are an emergent event, this means that they cannot be made to happen but at best, might be occasioned (Davis, Sumara, & Kieren, 1996). In this particular context, even though we as teachers, planned the lesson in ways that were conducive to personal and collective activity, this was no guarantee that the latter would happen and in actuality, when the lesson unfolded, it was not a case of either-or but rather, both(!) in which the absence and the emergence of a collective occurred.

In relation to the fact that it is impossible to predict whether a collective will form or what it will entail when it does because collectives are contingent on the moment-to-moment responses of the individual students that create it, by comparing the conceptual incoherence that resulted through the transactions that took place between Maeve and her peers’ with the coherence and collectivity that emerged for Sonia and Jade, we can appreciate the complementary impact that the two different patterns of flow had on the events of the lesson. Taking on yet a different stance, we can also recognize the absence of collectivity among Maeve and her peers’ as playing an important and *necessary* role in occasioning the emergence of collectivity of Sonia and Jade. For instance, had Maeve,

Sonia, and Jade all created identical models of the mystery object, it is entirely possible that there would have been no need for Sonia and Jade to form a collective and the three students could have continued to work individually on the task to justify why they all thought the object had the same identity. What happened instead, was a recurring neighboring interaction and consequent generative friction that arose when Maeve's conviction that the object could only be a cube came in contact with Sonia and Jade's conviction that the object had to be rectangular prism with two squares and four oblongs.

This conceptual friction, we contend, provoked the recurrent transactions among the three students and afforded several opportunities in which Jade and Sonia coordinated and re-coordinated their personal conceptions, eventually cohering as a collective. Recursively for us, this insight enables us to elaborate on our previous assertion that "collectives are contingent on the moment-to-moment responses of the individual students that create it." It is important to note that although Maeve did not become a member of the collective, the generative friction that prompted the collective was a co-creation of her personal conceptions and those of Sonia and Jade's. As such, Maeve's conceptualizations most definitely played an essential role in the emergence of the collective between Sonia and Jade.

Finally, previous classroom studies that examine the collective activity of students predominantly focus on the growth of understanding that evolves within the collective for the collective. This research and its continuation in a different manner points to how a student collective through the coordination and re-coordination of their pedagogical actions prompt new experiences and conceptualizations for another student who is not part of the collective.

### **Conclusion**

In this paper, we observed and inquired into the moment-to-moment transactions of three elementary students as they worked to identify an object they could feel but not see. Through the lens of complexity science, our interpretations of the students' conceptualizations and actions revealed two distinct patterns of flow that eventuate both the absence of collectivity and the emergence of it. Importantly, by studying the co-existence of these events, we gained an understanding of how the absence of one collective played an important and necessary role in occasioning another collective.

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<sup>2</sup> Descriptions for these appear in the transcript as text inside square brackets.